S-plane Explained

A (P)REVIEW OF LAPLACE TRANFORMS AND HOW TO USE THEM IN EE122

- Introduction
- Motivation for s-plane analysis
- When I say ‘s-plane,’ what do I mean?
- How to <u>use</u> the s-plane
  - Circuit analysis/design: the Integrator
  - Signal analysis/design: FM

Motivation for s-plane analysis

- Understanding circuits in another "domain" (the frequency domain)
  - The more ways we can understand our circuit, the better because...
- The time domain is hard for some things
  - filtering = convolution (Doh!)
- The s-plane (often) makes things easier
  - filtering = multiplication (Woohoo!)

Math behind the s-plane

The Laplace Transform of a time-domain function gives us a function of the complex frequency variable 's'

- The actual Laplace Transform*:

$$H(s) = \int_0^{\infty} h(t) e^{-st} dt$$

  - h(t) could be a signal, or a system
    - As a signal: $h(t) = \sin(15t + 0.16)$
    - As a system: $h(t) = \dfrac{v_{out}(t)}{v_{in}(t)}$
  - H(s): Laplace Transform of h(t)
  - s = σ + jω: Complex Frequency Variable

*: not necessary for EE122, but interesting (maybe)

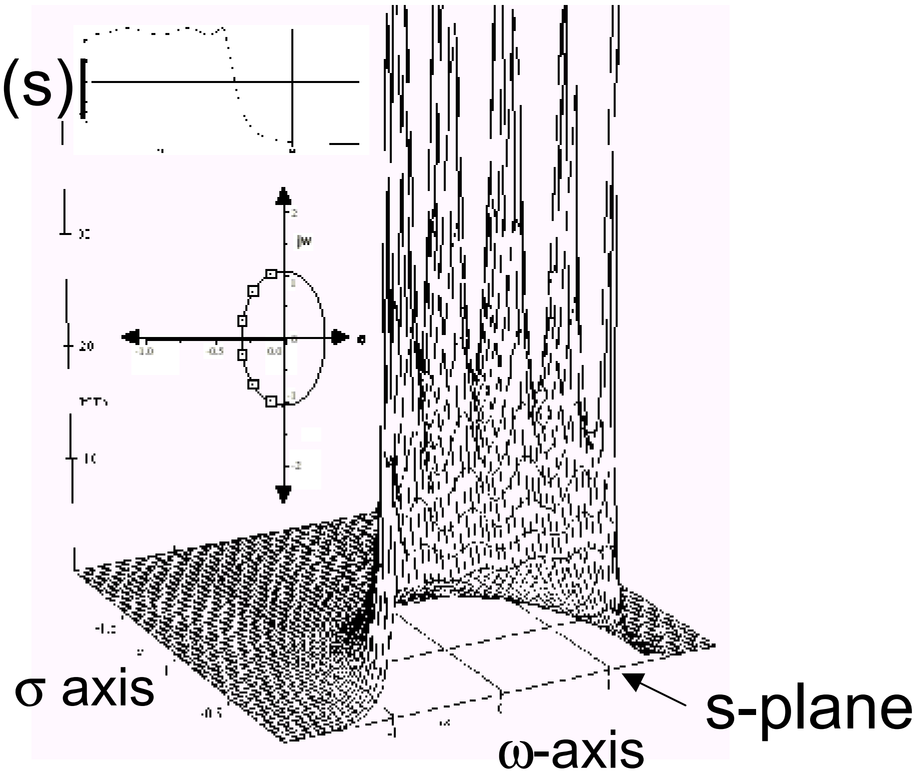
A note about transforms

- They make life easier (or should)
- If you’ve taken E40 (and/or Math 53), you have already used transforms
  - E40 uses phasor analysis of linear circuits
    - Complex impedances are just a tricky way of getting around differential equations so that we can take C as a “resistor” of value 1/ $j\omega C$. Indeed, they are simply Laplace (or Fourier) transforms in disguise--as you will see.
  - Math 53 uses Laplace Transforms to solve differential equations

What do I mean when I say 's-*plane*?'

ANSWER: THE REAL (σ) AND IMAGINARY (ω) PARTS OF s ARE THE 'x' and 'y' ON A GRAPH

- Example:
  - "Tent pole" plot of the magnitude of a Chebychev filter's response to inputs of complex frequency 's'
  - ω is what you usually think of as 'radian frequency' ($\omega=2\pi f$)
  - σ is decay rate*



*: Not important for EE122, but used in stability analysis

Why should I use the s-plane?

THIS IS THE SAME QUESTION AS:
'WHY IS THE LAPLACE TRANSFORM USEFUL?'

- Turns differential equations (hard) into algebraic equations (easy)

$$A v_{out}(t) + B\, dv_{out}/dt = C v_{in}(t) \Leftrightarrow (A + sB) v_{out} = C v_{in}$$

$$\frac{v_{out}}{v_{in}} = H(s) = \frac{C}{A + sB}$$

- Turns convolution (hard) into multiplication (easy)

$$f(t) * h(t) = g(t) \Leftrightarrow F(s) H(s) = G(s)$$

How to use s-plane analysis

- Examples to follow:
  - Systems
    - Simple RC circuit
    - Integrator
  - Signals
    - Multiplication by a sinusoid: “Heterodyning”

RC circuit (simple)

- Straight from EE122 lecture notes:

Derivation:

$$v_{out}(t) = v_{in}(t) - iR$$

$$i = C\, dv_{out}/dt$$

$$v_{out}(t) + RC\, dv_{out}/dt = v_{in}(t)$$

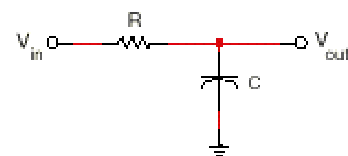
$$\text{"}Magic\text{" } (Laplace\ Transform)$$

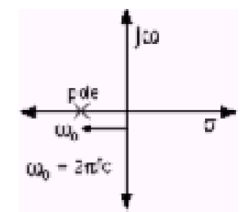
$$v_{out}(1 + sRC) = v_{in}$$

$$\frac{v_{out}}{v_{in}} = H(s) = \frac{1}{1 + sRC}$$

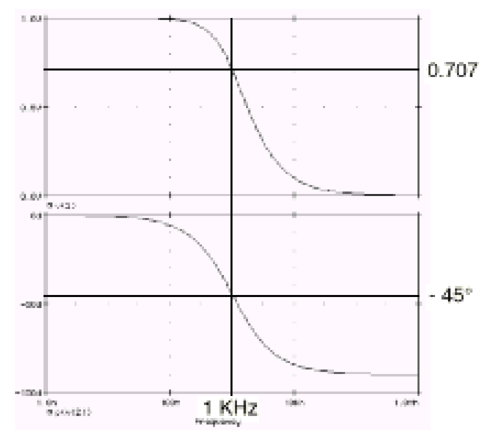
REVIEW: FIRST ORDER LOW-PASS

$$\frac{V_{out}}{V_{in}} = \frac{a_0}{S + \omega_0}$$





$$\omega_0 = 2\pi f_c = \frac{1}{RC}$$



Integrator (slightly less simple)

$$AC\ Analysis:$$

$$v_{out}(t) = -(i_{in}(t) - i_C(t))R_2$$

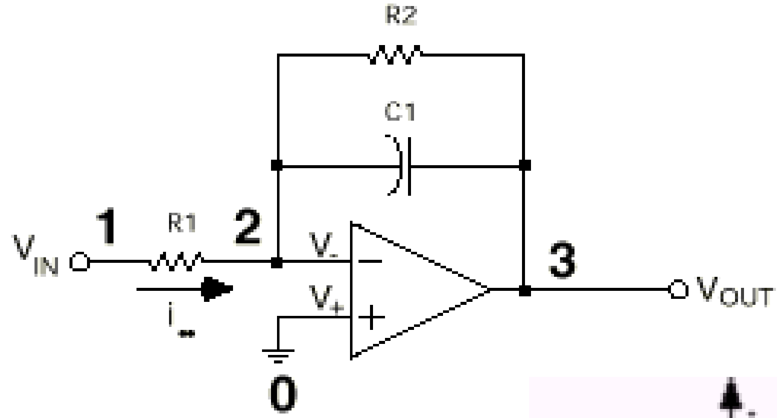
$$i_{in}(t) = v_{in}(t)/R_1$$

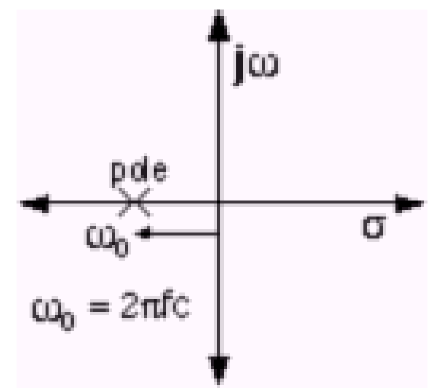
$$i_C(t) = -Cdv_{out}/dt$$

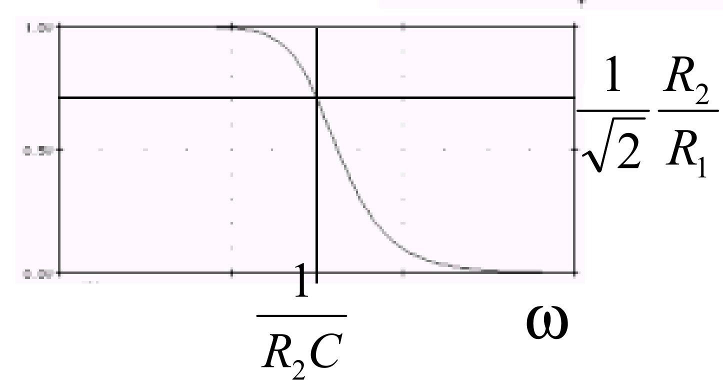
$$v_{out}(t) = -(v_{in}(t)/R_1 + Cdv_{out}/dt)R_2$$

$$(R_1 + sR_1R_2C)v_{out} = -v_{in}R_2$$

$$\frac{v_{out}}{v_{in}} = |H(s)| = \frac{R_2}{sR_1R_2C + R_1}$$



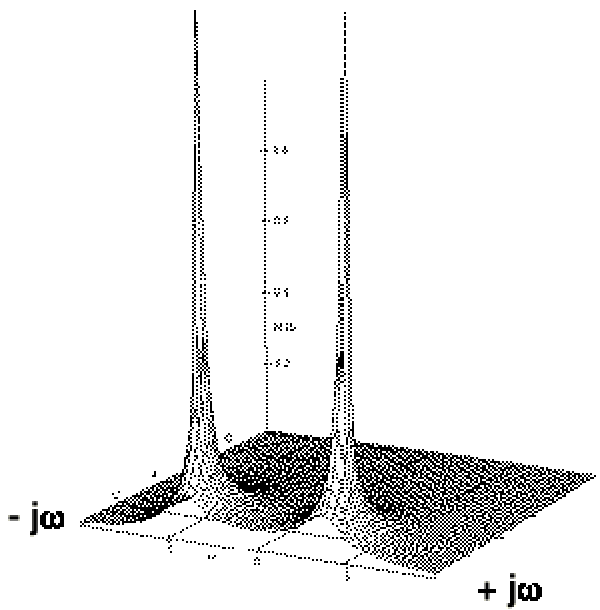




A familiar slide

POLES, ZEROS, AND CIRCUS TENTS?

$$T(S) = \frac{A(S)}{B(S)} = \frac{(S - Z_1)(S - Z_2)\cdots(S - Z_m)}{(S - P_1)(S - P_2)\cdots(S - P_n)}$$

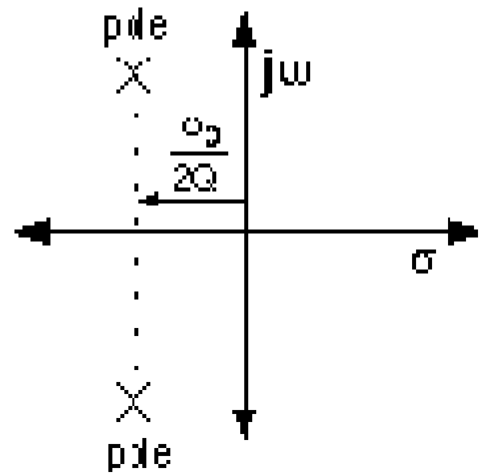


$$LPF(S) = \frac{A}{[S + (1+4j)][S + (1-4j)]}$$

Another familiar slide...

POLES OF 2nd ORDER SYSTEMS

$$LPF(S) = \frac{A}{S^2 + S\frac{\omega_o}{Q} + \omega_o^2}$$



POLE DISTANCE FROM j ω AXIS = $\frac{\omega_o}{2Q}$

FOR LEAST PEAKING ("MAXIMALLY-FLAT")

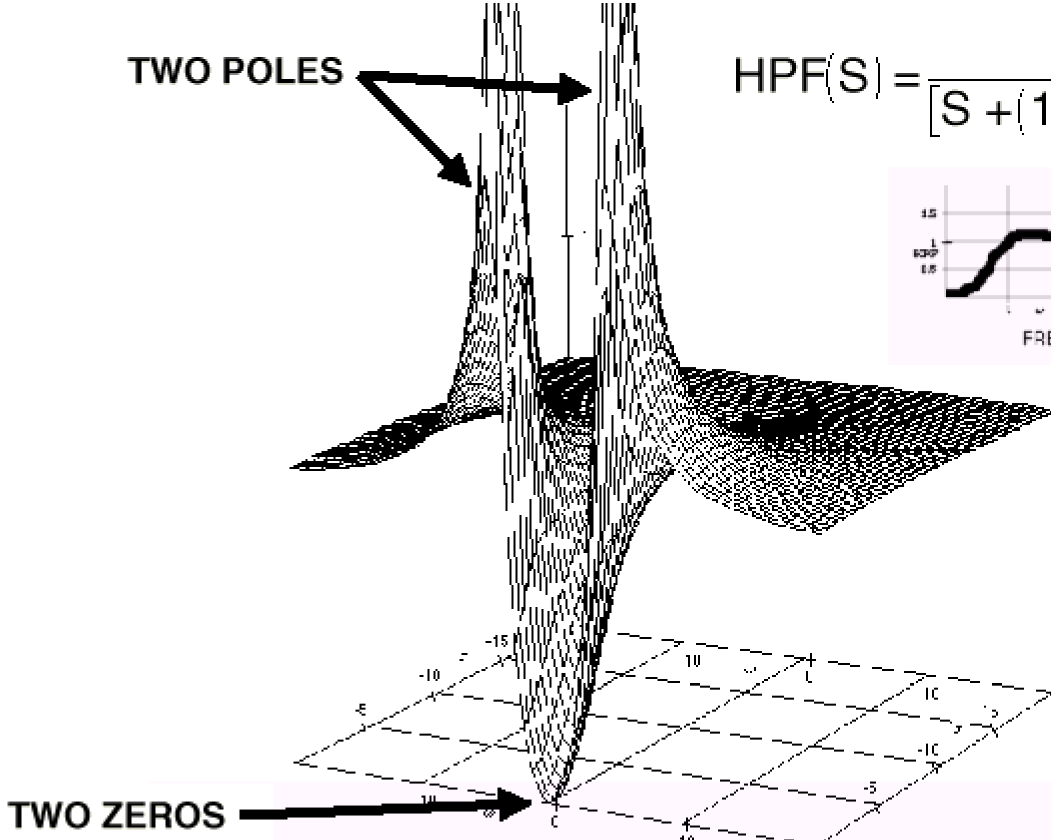
$$Q = \frac{1}{\sqrt{2}}$$

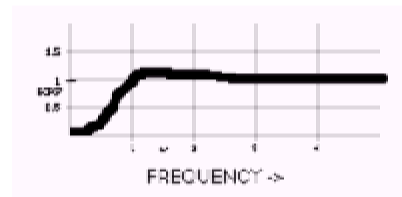
$$P_{1,2} = -\frac{\omega_o}{2Q} \pm j\omega_o\sqrt{1 - \frac{1}{(2Q)^2}}$$

A last familiar slide

EXAMPLE: 2nd ORDER HIGH-PASS

$$HPF(S) = \frac{S^2}{[S + (1+4j)][S + (1-4j)]}$$





Heterodyning (for FM radio)

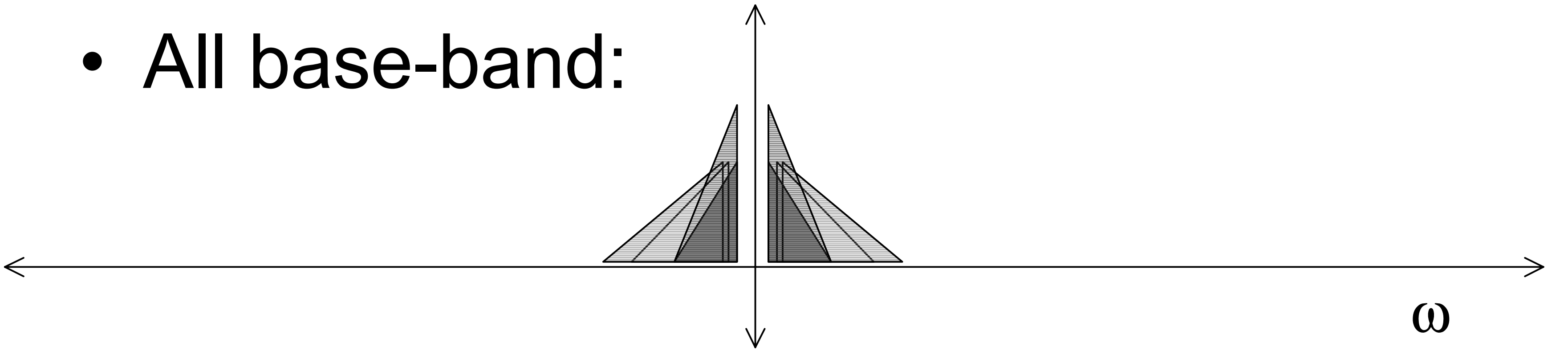
- Motivation: “Base-band” txn is trouble
  - Efficiency
    - $\lambda/4$ Antenna for kHz ~ length > kilometers
  - Interference
    - All of the signals would muddle together
  - Hence, we need distinct ‘carrier frequencies’ (regulated by the FCC)
    - 92.3 MHz (KSJO)
    - 88.5 MHz (KQED)

      FM
    - 1070 kHz (KNX)
    - 710 kHz (KABC)

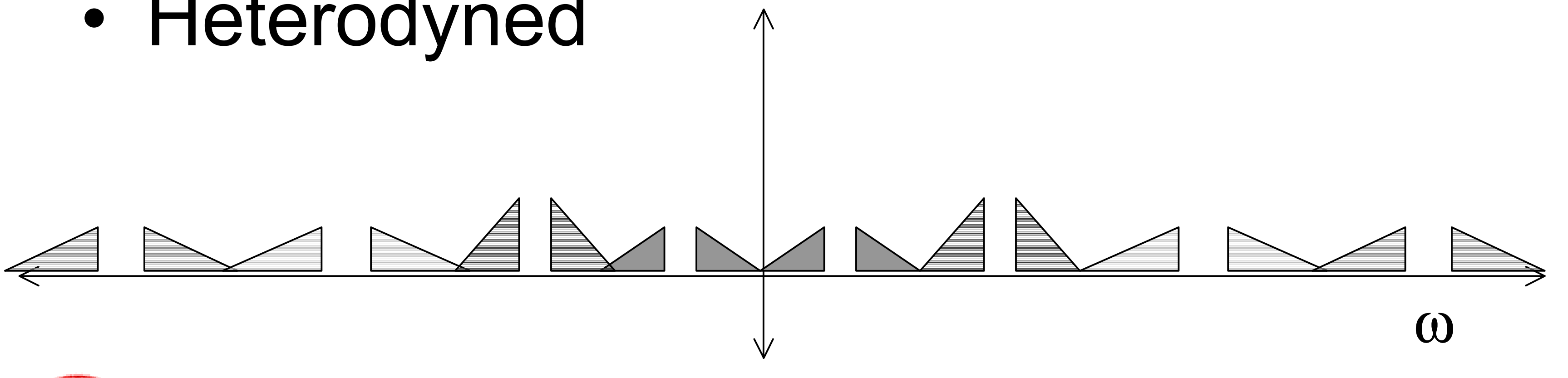
      AM

The "frequency domain"

- All base-band:

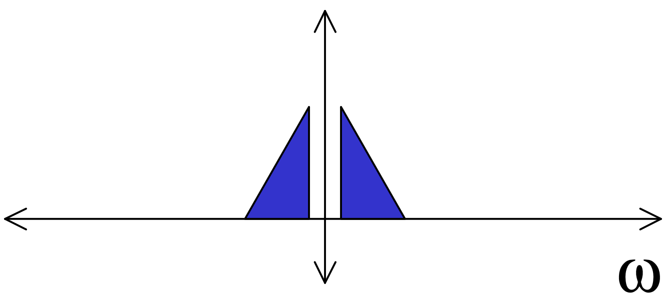


- Heterodyned

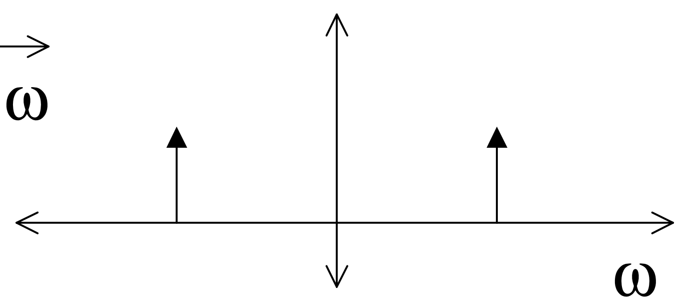


Back to heterodyning

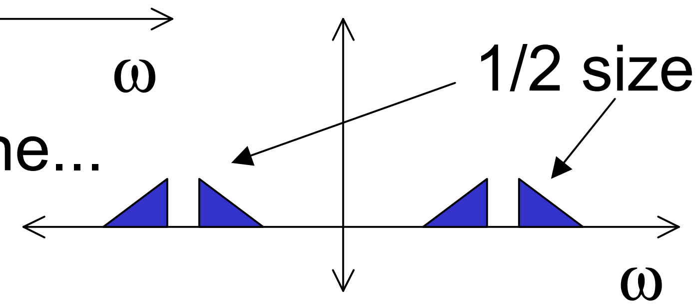
Because convolution in the time domain is multiplication in the frequency domain, when we multiply by a sinusoid in the time domain, it convolves the frequency content of our signal up to the frequency of the sinusoid.
(Whew! That was a mouthful)



Base-band signal...



…convolved with pure tone...



…yields heterodyned signal!